



John Wollaston Anglican Community School

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

SOLUTIONS

Section Two: Calculator-assumed

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

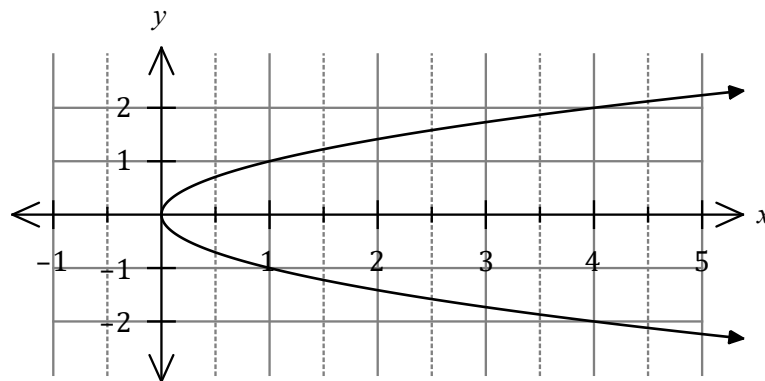
This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(6 marks)

(a) The parabolic graph of a relation is shown below.



(i) State the equation of its axis of symmetry. (1 mark)

Solution
$y = 0$
Specific behaviours
✓ correct equation

(ii) State the equation of the relationship between x and y . (1 mark)

Solution
$x = y^2$
Specific behaviours
✓ correct equation

(b) Points A and B have coordinates $(-2, 4)$ and $(-6, 10)$ respectively. Determine the equation of the circle that has diameter AB . (4 marks)

Solution
Midpoint: $\left(\frac{-2 - 6}{2}, \frac{4 + 10}{2}\right) = (-4, 7)$
Radius: $r^2 = (-6 + 4)^2 + (10 - 7)^2$ $= 13$
Equation: $(x + 4)^2 + (y - 7)^2 = 13$
Specific behaviours
<ul style="list-style-type: none"> ✓ identifies midpoint as centre ✓ calculates coordinates of centre ✓ calculates square of radius ✓ writes equation of circle

Question 10**(6 marks)**

A survey of all apartments for sale in a particular suburb showed that 54% had a lift and 60% had secure parking. Of those with secure parking, 75% had a lift.

Determine the probability that a randomly selected apartment from those in the survey had

- (a) a lift and secure parking. (2 marks)

Solution
$P(L \cap S) = 0.6 \times 0.75$ $= 0.45$
Specific behaviours
✓ indicates correct method ✓ calculates probability

- (b) a lift or secure parking. (2 marks)

Solution
$P(L \cup S) = 0.54 + 0.60 - 0.45$ $= 0.69$
Specific behaviours
✓ indicates correct method ✓ calculates probability

- (c) no lift or no secure parking. (2 marks)

Solution
$P(\bar{L} \cup \bar{S}) = 1 - P(L \cap S)$ $= 1 - 0.45$ $= 0.55$
Specific behaviours
✓ indicates correct method ✓ calculates probability

Question 11

(8 marks)

- (a) Triangle ABC is such that $b = 15$ cm, $c = 18$ cm and $\angle A = 125^\circ$. Determine, with justification, the length of side a . (2 marks)

Solution
$a^2 = 15^2 + 18^2 - 2(15)(18) \cos 125^\circ$ $a = 29.3 \text{ cm}$
Specific behaviours
<ul style="list-style-type: none"> ✓ clearly shows use of cosine rule ✓ correct length

- (b) Triangle PQR is such that $p = 48.1$ cm, $q = 41.5$ cm and $\angle Q = 45^\circ$. Determine all possible areas of this triangle. (6 marks)

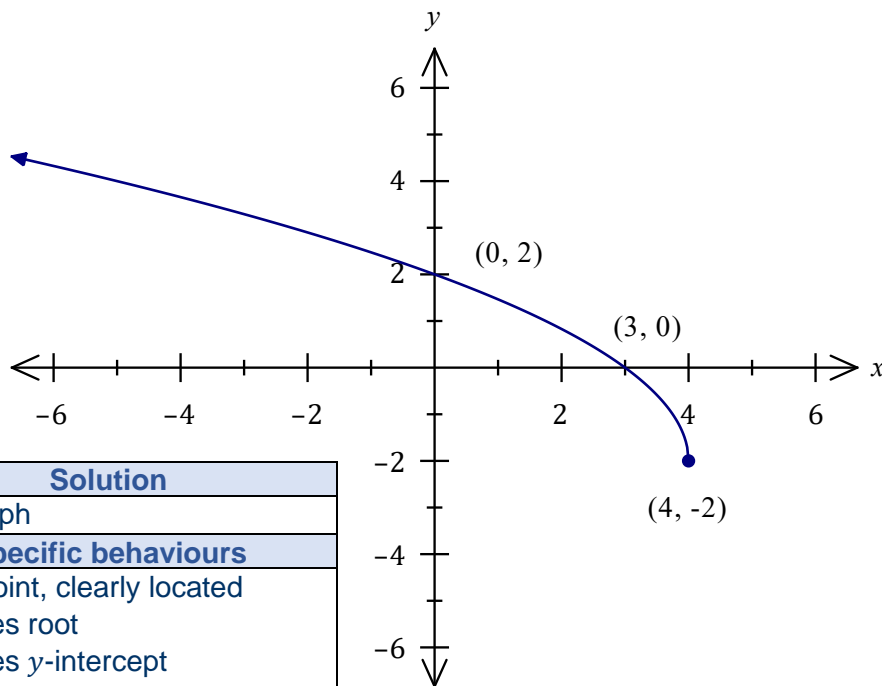
Solution
$\frac{\sin P}{48.1} = \frac{\sin 45^\circ}{41.5}$
<p>First solution:</p> $\angle P_1 = 55^\circ$ $\angle R_1 = 180^\circ - 45^\circ - 55^\circ = 80^\circ$ $A_1 = \frac{1}{2}(48.1)(41.5) \sin 80^\circ$ $A_1 = 983 \text{ cm}^2$
<p>Second solution:</p> $\angle P_2 = 180^\circ - 55^\circ = 125^\circ$ $\angle R_2 = 180^\circ - 45^\circ - 125^\circ = 10^\circ$ $A_2 = \frac{1}{2}(48.1)(41.5) \sin 10^\circ$ $A_2 = 174 \text{ cm}^2$ <p style="text-align: center;">Areas are 174 cm^2 and 983 cm^2.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ shows use of sine rule ✓ calculates one value for $\angle P$ ✓ derives $\angle R$ and shows use of area formula ✓ calculates one correct area ✓ calculates second set of values for $\angle P$ and $\angle R$ ✓ calculates second area

Question 12

(8 marks)

Let $f(x) = 2\sqrt{4-x} - 2$.(a) Sketch the graph of $y = f(x)$ on the axes below.

(4 marks)



Solution
See graph
Specific behaviours
<ul style="list-style-type: none"> ✓ endpoint, clearly located ✓ locates root ✓ locates y-intercept ✓ smooth curve, no inflections

(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of $y = f(x)$:

(i) $y = 2\sqrt{1-x} - 2$.

(2 marks)

Solution
$y = f(x - 3)$. Horizontal translation of 3 units to the right.
Specific behaviours
<ul style="list-style-type: none"> ✓ states a translation ✓ correct distance and direction

(ii) $y = \sqrt{4-x} - 1$.

(2 marks)

Solution
$y = \frac{1}{2}f(x)$. Vertical dilation of scale factor $\frac{1}{2}$.
Specific behaviours
<ul style="list-style-type: none"> ✓ both bolded words in description ✓ correct scale factor

Question 13

(8 marks)

A study of the achievements of 250 students enrolled in a university course yielded the following information:

- 44.8% of all students did not achieve a distinction
- 25% of those who did not achieve a distinction studied full-time
- 60% of those who studied part-time did not achieve a distinction

(a) Use the above information to complete the following table. (4 marks)

	Part-time	Full-time	Totals
Distinction	56	82	138
No distinction	84	28	112
Totals	140	110	250

Solution			
$250 \times 0.448 = 112, \quad 112 \times 0.25 = 28, \quad 112 - 28 = 84$			
$\frac{84}{x} = 0.60 \Rightarrow x = 140, \quad 140 - 84 = 56, \quad 250 - 112 = 138, \quad 138 - 56 = 82$			
Specific behaviours			
✓ splits total to D/ND; ✓ splits no distinction to FT/PT			
✓ calculates PT total; ✓ completes table			

(b) Determine the probability that a randomly chosen student from the study

(i) studied part-time and achieved a distinction. (1 mark)

Solution
$P = \frac{56}{250} = \frac{28}{125} = 0.224$
Specific behaviours
✓ correct probability

(ii) studied part-time or achieved a distinction. (1 mark)

Solution
$P = \frac{250 - 28}{250} = \frac{222}{250} = \frac{111}{125} = 0.888$
Specific behaviours
✓ correct probability

(c) Explain whether this study provides any evidence that achieving a distinction is independent of study mode. (2 marks)

Solution
$P(D) = 1 - 0.448 = 0.552$
$P(D PT) = 1 - 0.60 = 0.40$
Hence events are not independent as $P(D) \neq P(D PT)$.
Specific behaviours
✓ states $P(D)$ and a conditional probability
✓ states not independent with reason

Question 14

(9 marks)

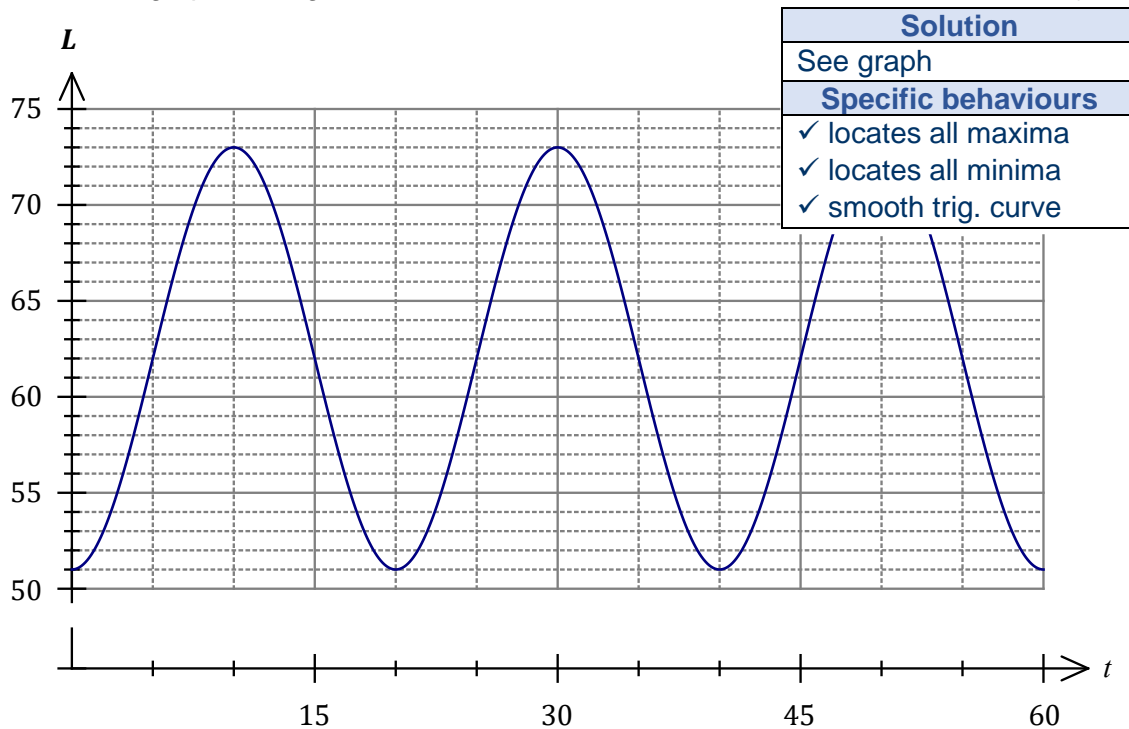
The loudness L of sound, in decibels, emitted by a machine t minutes after it is switched on can be modelled by

$$L = 62 - 11 \cos\left(\frac{\pi t}{10}\right)$$

- (a) Determine the initial loudness emitted by the machine. (1 mark)

Solution
$L(0) = 51$ dB
Specific behaviours
✓ correct value

- (b) Draw the graph of L against t on the axes below for the first 60 minutes. (3 marks)



Solution
See graph
Specific behaviours
✓ locates all maxima
✓ locates all minima
✓ smooth trig. curve

- (c) State the maximum loudness emitted by the machine and the time this maximum was first reached. (2 marks)

Solution
$L_{MAX} = 73$ dB when $t = 10$ s.
Specific behaviours
✓ correct maximum
✓ correct time

- (d) A health and safety inspector can deem a machine unserviceable if the loudness it emits exceeds 70 dB for more than 15 minutes in any hour that it is running. Determine, with justification, whether this machine could be deemed unserviceable. (3 marks)

Solution
Exceeds value for $7.59 < t < 12.41$ during first cycle.
$3(12.41 - 7.59) = 14.5$ minutes per hour - and so machine could not be deemed unserviceable.
Specific behaviours
✓ identifies interval endpoints
✓ calculates minutes per hour
✓ uses calculations to draw conclusion

See next page

Question 15

(8 marks)

A souvenir shop sells T-shirts in two colours and three sizes. Sales records for the past year are shown below.

	Small	Medium	Large
White	280	370	200
Black	250	310	190

Assume that the shop holds a large stock and that sales continue in similar proportions. Where relevant, round your answers in this question to three decimal places.

(a) A customer randomly selects a T-shirt for purchase. Determine

(i) the most likely size and colour of this T-shirt and the probability that this T-shirt is selected. (2 marks)

Solution
Most likely: White, medium. $P = \frac{370}{1600} = \frac{37}{160} \approx 0.231$
Specific behaviours
<ul style="list-style-type: none"> ✓ type of T-shirt ✓ calculates probability

(ii) the probability that the T-shirt selected is not a medium. (2 marks)

Solution
$P = \frac{530 + 390}{1600} = \frac{920}{1600} = \frac{23}{40} = 0.575$
Specific behaviours
<ul style="list-style-type: none"> ✓ counts required sizes ✓ calculates probability

(b) A customer randomly selects two T-shirts for purchase. Determine the probability that the T-shirts are

(i) both large. (2 marks)

Solution
$P(LL) = \left(\frac{390}{1600}\right)^2 = \frac{1521}{25600} \approx 0.059$
Specific behaviours
<ul style="list-style-type: none"> ✓ probability of one large ✓ calculates probability

(ii) of different colours. (2 marks)

Solution
$P(WB) = \frac{850}{1600} \times \frac{750}{1600} = \frac{255}{1024} (\approx 0.249)$
$\therefore P(WB \cup BW) = 2 \times \frac{255}{1024} = \frac{255}{512} \approx 0.498$
Specific behaviours
<ul style="list-style-type: none"> ✓ probability of one then the other ✓ calculates probability

Question 16

(8 marks)

- (a) Let $f(x) = x^2 + bx + c$, where b and c are constants. The graph of $y = f(x)$ has an axis of symmetry with equation $x = -3$ and an axis intercept at $(0, 5)$.

- (i) State the value of the constant c . (1 mark)

Solution
c is the y -intercept: $c = 5$.
Specific behaviours
✓ correct value

- (ii) Determine the value of the constant b . (2 marks)

Solution
Axis of symmetry has equation $x = -\frac{b}{2a}$: $-3 = -\frac{b}{2} \Rightarrow b = 6$
Specific behaviours
✓ indicates appropriate method ✓ calculates value

- (b) Let $g(x) = 2(x - 2)^2 - 7$. Determine

- (i) the coordinates of the turning point of the graph of $y = g(x)$. (1 mark)

Solution
Turning point is at $(2, -7)$.
Specific behaviours
✓ correct coordinates

- (ii) the domain and range of $g(x)$. (2 marks)

Solution
Domain: $x \in \mathbb{R}$, and range: $y \geq -7$.
Specific behaviours
✓ states domain ✓ states range

- (iii) the coordinates of the turning point of the graph of $y = g(x - 3) + 2$. (2 marks)

Solution
Graph has been translated 3 units right and 2 units upwards and so new turning point at $(5, -5)$.
Specific behaviours
✓ indicates correct use of one translation ✓ correct coordinates

Question 17

(8 marks)

A chess club has 12 members, of which 5 are beginners, 3 are intermediate and the rest are advanced. The club has to select a group of 4 members at random to assist with a regional tournament.

(a) Determine the number of different groups that can be selected.

(2 marks)

Solution
$\binom{12}{4} = 495$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly uses any combination notation ✓ correct number

(b) Determine the number of different groups that can be selected which contain at least 3 beginners.

(2 marks)

Solution
Choose 3 beginners: $\binom{5}{3} \binom{7}{1} = 70$ Choose 4 beginners: $\binom{5}{4} \binom{7}{0} = 5$ Total: $70 + 5 = 75$
Specific behaviours
<ul style="list-style-type: none"> ✓ number with 3 beginners ✓ number with 4 beginners and total

(c) Determine the probability that the group contains

(i) no advanced members.

(2 marks)

Solution
$\binom{8}{4} \binom{4}{0} = 70 \rightarrow P = \frac{70}{495} = \frac{14}{99} = 0.\overline{14}$
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates number ✓ states probability (no need to simplify)

(ii) exactly one intermediate member.

(2 marks)

Solution
$\binom{3}{1} \binom{9}{3} = 252 \rightarrow P = \frac{252}{495} = 0.509$
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates number ✓ states probability (no need to simplify)

Question 18

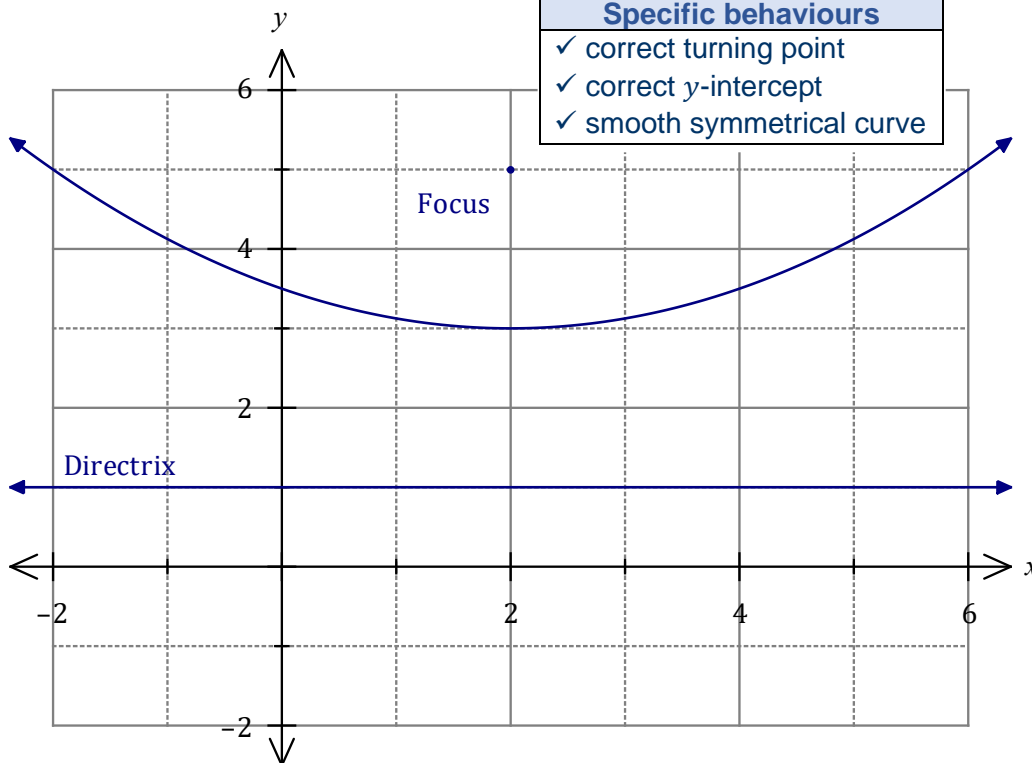
(7 marks)

The equation of a parabola is $y = \frac{1}{8}(x^2 - 4x + 28)$.

(a) Sketch the parabola on the axes below.

Solution
See graph
Specific behaviours
✓ correct turning point
✓ correct y-intercept
✓ smooth symmetrical curve

(3 marks)



All parabolas have a focal point and a directrix. For a parabola with equation $y = a(x - p)^2 + q$, the focal point is at $(p, q + \frac{1}{4a})$ and the equation of the directrix is $y = q - \frac{1}{4a}$, where a, p and q are constants.

(b) Determine the focal point and directrix for this parabola and add them, with labels, to your sketch above. (4 marks)

Solution
From graph, turning point at (2, 3). Hence
$a = \frac{1}{8}, \quad p = 2, \quad q = 3$
Focal point: (2, 5) and directrix: $y = 1$.
Specific behaviours
✓ indicates turning point
✓ indicates values of all constants
✓ plots focus
✓ draws directrix

Question 19

(8 marks)

The events S and T are such that $P(S) = 0.4$ and $P(T) = 0.3$.

Determine $P(S \cup T)$ in each of the following cases:

(a) S and T are mutually exclusive.

(1 mark)

Solution
$P(S \cup T) = 0.4 + 0.3 = 0.7$
Specific behaviours
✓ correct probability

(b) $P(\bar{S} \cup T) = 0.85$.

(2 marks)

Solution
$P(S \cap \bar{T}) = 1 - 0.85 = 0.15$
$P(S \cup T) = 0.3 + 0.15 = 0.45 = \frac{9}{20}$
Specific behaviours
✓ calculates $P(S \cap \bar{T})$
✓ calculates probability

(c) S and T are independent.

(2 marks)

Solution
$P(S \cap T) = 0.4 \times 0.3 = 0.12$
$P(S \cup T) = 0.4 + 0.3 - 0.12 = 0.58 = \frac{29}{50}$
Specific behaviours
✓ calculates $P(S \cap T)$
✓ calculates probability

(d) $P(T|\bar{S}) = 0.25$.

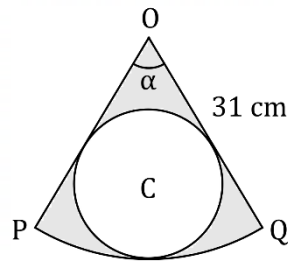
(3 marks)

Solution
$P(T \bar{S}) = \frac{P(T \cap \bar{S})}{P(\bar{S})}$
$P(T \cap \bar{S}) = (1 - 0.4) \times 0.25 = 0.15$
$P(S \cup T) = 0.4 + 0.15 = 0.55 = \frac{11}{20}$
Specific behaviours
✓ derives conditional probability relationship
✓ calculates $P(T \cap \bar{S})$
✓ calculates probability

Question 20**(7 marks)**

The diagram shows sector OPQ of a circle centre O of radius 31 cm and $\alpha = 40^\circ$.

Circle C is inside the sector and just touches OP , OQ and arc PQ .



- (a) Determine the area of sector
- OPQ
- .

(2 marks)

Solution
$A = \frac{40^\circ}{360^\circ} \times \pi(31)^2 = \frac{961\pi}{9} \approx 335 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates suitable method ✓ calculates area

- (b) Show that the radius of circle
- C
- is 7.9 cm, correct to one decimal place.

(3 marks)

Solution
Extend sector sides to form isosceles triangle:
$MQ' = 31 \tan\left(\frac{40^\circ}{2}\right) = 11.28 \text{ cm}$
$r = 11.28 \tan\left(\frac{70^\circ}{2}\right) = 7.9 \text{ cm}$
Specific behaviours
<ul style="list-style-type: none"> ✓ forms isosceles triangle, uses half-angle ✓ calculates MQ' ✓ calculates radius

- (c) Determine the area of the shaded region, inside sector
- OPQ
- but outside circle
- C
- .

(2 marks)

Solution
$A_C = \pi(7.9)^2 \approx 196$
$\text{Shaded area} = 335 - 196 = 139 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates area of circle ✓ calculates shaded area, with units

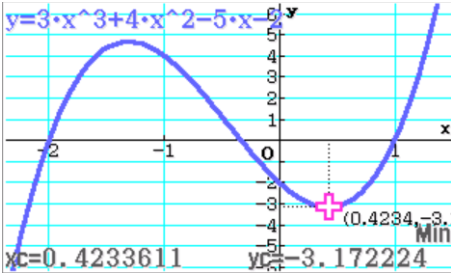
Question 21

(7 marks)

The equation $f(x) = k$ has just one solution, where $f(x) = ax^3 + bx^2 - 5x - 2$, and a, b and k are constants.

The graph of $y = f(x)$ cuts the x -axis at $x = 1$, $x = -2$, and at one other point.

Determine the value(s) of the constant k , rounded to 2 decimal places. Explain your reasoning.

Solution
<p>Use roots to solve for a and b:</p> $f(1) = 0 \Rightarrow a + b - 5 - 2 = 0$ $f(-2) = 0 \Rightarrow -8a + 4b + 10 - 2 = 0$ <p>Solving simultaneously with CAS gives $a = 3$ and $b = 4$.</p>  <p>For one solution, k must be greater than the local maximum or less than the local minimum of $f(x)$ - found using CAS.</p> <p>Local maximum is $y = 4.6702$</p> <p>Local minimum is $y = -3.1722$</p> <p>Hence $k < -3.17$ or $k > 4.67$.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates solving for a and b ✓ equates $f(1) = 0, f(-2) = 0$ ✓ identifies equations as simultaneous ✓ solves equations for a and b ✓ describes case for one solution ✓ states value of local minimum, maximum ✓ correct inequalities for k

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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